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Preliminaries to any Theory of Representation

In contemporary writings on cognition, problem solving, and intellectual development the expressions "representation", "to represent something", and "to represent to oneself" crop up with a certain frequency. People seem to be quite comfortable with them and writers of the most divergent schools of thought use them confidently as though there should be no difficulty at all about their interpretation. Yet, one does not have to look very far in ordinary English texts to find occurrences of the verb "to represent" that show that the word is used with a rather wide range of meanings. Thus, when it is used in technical contexts but without a specific definition, it tends to remain opaque.

Because Goethe was brought into the discussion, I could not help but recall what his Mephistopheles, full of irony, says to the eager student: "Denn eben, wo Begriffe fehlen, da stellt ein Wort zur rechten Zeit sich ein." (In colloquial translation: "Just where we have no concepts, words come in very handy".)

If, eventually, we want to formulate a Theory of Representation, it would seem indispensable that, at the outset, we clarify as best we can what kind of conceptual structure we have in mind when we say "representation".

In the pages that follow I lay out some thoughts in that direction, thoughts that, inevitably, are determined by my conviction that there can be no viable theory of representation without an explicit theory of knowledge. Given the work I have been involved in during the last couple of decades, I am strongly biased in favor of a constructivist epistemology; and these brief notes, therefore, should be taken as an attempt to approach the problem of representation from that particular perspective

Four Distinctions

My starting point is perhaps best characterized by saying: "A representation does not represent by itself — it needs interpreting and, to be interpreted, it needs an interpreter."

Even a picture is not a picture *of* anything until a viewer (observer, experiencer) relates the colors and shapes he or she perceives in it to the record of things constructed in prior experience. I will interpret a photograph as a picture of myself, *if and only if* I perceive it as similar to images I have at other times perceived when looking into a mirror. But interpretation may also proceed by combining parts. The

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monsters, for instance, on a canvas by Max Ernst Hieronymus Bosch do not represent "things" in anyone's experiential world. They would be better characterized as presentations of originals that these somewhat unusual men cooked up in their imagination, and these presentations are perceived by us as monsters, only because we have experienced lips. eyes breasts, beaks, claws, and feathers in our own time and are now led to combine these items in novel, unexpected ways. As such, these paintings do not *re*-present but simply provide an occasion to construct something new, and it is up to us, the viewers, to interpret the presentation as creatures of HeII, the Garden of Eden, or caricatures of everyday life.

In discussing representation it seems even easier than in other contexts to produce nonsense. The situation is particularly complicated because the word "representation" is fraught with ambiguity that, for the most part, remains hidden and thus creates untold conceptual confusion. As so often, however, ambiguities surface and become quite obvious when we translate into another] language. In the case of representation, I know no better way to lay out the conceptual mess than to escape into German, because there, in different contexts, different words are needed to translate what would be covered by "to represent" in English.

German, in fact, keeps apart the following principal meanings that, in English, are compounded in one word (further distinctions could be made. but for the purpose of this exposition these four suffice):

1. The sketch represents (depicts) a lily	= darstellen
2. Jane ("mentally") represents something to herself	= vorstellen
3. Mr.Bush represents (acts for) the president	= vertreten
4. "X" represents (stands for, signifies, denotes) some	= bedeuten
unknown guantity	

Given that the German words are usually not interchangeable, there are obviously conceptual differences that, whether he likes it or not, the German speaker is compelled to keep apart. Because many of the problems we meet in our discussions seem to have something to do with this ambiguity, it would seem helpful to try to separate as neatly as possible the different conceptual structures that become confounded in the fuzzy term "representation".

The conceptual maze is even more involved than the fourfold division suggests. In sentence (1), "the sketch" (the grammatical subject) refers to the item that *does* the representing. If this grammatical subject is replaced by "the artist", the meaning of "represents" does not change, although it is now not the item called "artist" that does the representing but a sketch or some such implied product of the artist's activity. This type of *agent/activity/product* ambiguity is common in our languages and it is not particularly relevant to the problem of representation. What is relevant is that an item such as a sketch, which the linguistic expression purports to be the active agent that does the representing (in the *darstellung/depiction* sense of "representation") is always the result of someone's productive activity. When we say "This sketch represents a lily", we are expressing a judgment about an experiential item that we have categorized as a sketch. Our judgment stems from this: Among the constituent operations, which we carry out spontaneously when we perceive (i.e., perceptually construct) this item, there are some that seem so similar to the operations that we spontaneously carry out when we perceive the kind of item we usually categorize as "a lily", that we are prepared to consider the two items equivalent *with respect to these particular perceptual operations*. (Needless to say, such a judgment will always be largely determined by the particular context in which we are operating.)

This, I believe, is compatible with Nelson Goodman's (1976) view. He says: "The making of a picture commonly participates in making what is to be pictured" (p.32).

Because perceiving, from a constructivist point of view, is always an active making, rather than a passive receiving, the similarity of a picture and what it depicts does not reside in the two objects but in the activities of the Experiencer who perceives them. Ordinary language, however, refers to objects as though they *existed* as such, independent of experience. Consequently, it always leads us, the language users, to attribute differences in our perceptual operating to the externalized objects as though they were properties belonging to them in an "objective" sense. Provided we remain aware of this epistemological sleight of hand, we may safely say: An iconic representation (Darstellung) is an artifact and a deliberate reconstruction of another experiential item; the reconstruction selects certain properties considered relevant under the circumstances and uses a medium different from the original.

From the constructivist perspective, the viewer's interpretation of an iconic representation, i.e., what the "icon" will be said to depict, cannot, as realists tend to believe, be a piece of the "external", ontic world, but only something that, under all circumstances, consists of elements that are already within the viewer's realm of experience. This is the aspect that is dealt with in the next section. In any case, then, such a representation is intended to stand for a previously constructed item, but it also differs in some way from that previous construction. It may be simplified or stylized, larger or smaller, two- instead of three-dimensional, or transformed in some other way.

A Darstellung or icon, then, has the specific function to "refer" to something else, another experiential item that it is supposed to depict.¹

Lest this be mistaken for a profession of "realism", let me once more emphasize the constructivist approach. A drawing, for instance, will be said to represent a lily, if it is able to produce in the experiencer a reconstruction of the kin of experience he or she has come to call a "lily". But this reconstruction must be somehow different from the construction that yields a "real" lily. If you perceived two lilies on the table, you would hardly consider one of them a representation of the other — in spite of the fact that, in order to be considered "lilies", they must both be constructed by you as the kind of experience that you have come to call "lily". The difference that leads one to distinguish a "real" lily from an iconic representation of a lily may, as I have suggested earlier, be of a variety of kinds, but the one crucial element is probably the realization that there are things that one can do with the lily but not with an iconic representation of one, and vice versa.

A *trompe l'oeil* painting of a lily, or a life-like lily made of plastic, may be intended to trick the beholder into mistaking it for the kind of experiential item that he would spontaneously call "a lily"; if this succeeds, he will say that it is a lily. and not merely that it is *like* a lily. In that case, he will assimilate the experiential item to his

¹ I use the term "icon" somewhat more loosely than did Peirce in his Theory of Signs (Collected Papers, 1931–1935, Vol. 2).

lily concept without becoming aware of the differences that make it an iconic representation. (Note: if that assimilation is deliberate, it may prompt one specific use of the word "but": e.g., "It's a lily, but a plastic one".)

Mental Representation

The difference between *darstellen* and *vorstellen* is in part analogous to the difference between a transitive and an intransitive activity. The first begins with a given object, the item that is to be depicted; the second does not begin with an object but creates one. As is so often the case with nouns, *Vorstellung* may refer to the activity or to the activity's result (compare "painting", "diversion", and, indeed, "representation"). In either case, however, *Vorstellung* or "mental representation" refers to a primary creation, to an act of perceptual or imaginal construction, and there is no prior object that serves as "original" to be replicated or *re*-presented.²

Hence it would be preferable to move the notion of Vorstellung altogether out of the semantic realm of "representation", but, given the present currency of that word, there is no hope that this could be generally accepted. However, in order to keep *mental* representations apart from the others, I accentuate their character of internal construction by referring to them as "conceptions".

In ordinary usage, the things we call "concept" (*Begriff*) often seem to coincide with Vorstellungen, but I would prefer to use the term concept for those conceptions that have been honed by repetition, standardized by interaction. and associated with a specific word. Both are, indeed, retrievable and thus repeatable; but each time one and the same conceptual item is presented, it is that item and not a copy or replica of it. In this context it is important to state that, in the constructivist view, "concepts", "mental representations", "memories". "images", etc. must not be thought of as static but always as dynamic; that is to say, they are not conceived as postcards that can be retrieved from some file, but rather as relatively self-contained programs or production routines that can be called up and run. Conceptions, then, are produced internally. They are replayed, shelved, or discarded according to their usefulness and applicability in experiential contexts. The more often they turn out to be viable, the more solid and reliable they seem. But no amount of usefulness or reliability can alter their internal, conceptual origin. They are not replicas of external originals, simply because no cognitive organism can have access to "things-in-themselves" and thus there are no models to be copied.

No matter how new it may seem, a conception is always made up of elements that first arose on the sensorimotor level of experience. Thus, they are made up of elements that the experiencing subject already has, though they may, of course, be novel combinations in the same sense that the visual image we construct when we

² This is one reason why Kant is so often misunderstood when read in English. In the first sentence of the *Introduction* to his *Critique of Pure Reason*, he rhetorically asks "how could the cognitive faculty be stirred into action, if not by objects which activate our senses and engender *Vorstellungen* ..." — If, as seems to be the rule, this last term is translated as "representations, the English reader is at once misled into believing that these *Vorstellungen* are to be misunderstood as some sort of pictures of objects. This notion will inevitably be reinforced by subsequent occurrences of representation in spite of the fact that it makes nonsense of Kant's theory of knowledge.

perceive a Max Ernst or Hieronymus Bosch painting is a novel structure in spite of the fact that it consists of well-known parts that we have often used before.

Moreover, these conceptions may, of course, exemplify some abstraction — but if they do, they do so by applying the abstraction to quite specific sensorimotor material. It is in this sense that Berkeley was right when he said that we could never visualize a "pure generalization" or a "universal" and that, therefore, no such abstraction could "exist". But what Berkeley did not consider was that we could very well retain the way of constructing them, the set of operations that constitute them, or, if you can accept that metaphor, the "program" that produces them. (One's *mental representation* of, say, one hundred will be either the numeral "100" or "C", or a specific lot of unitary items whose count is presumed to yield the number word "hundred", or an arrangement of specific lots according to a transform derived from the accepted symbol system, such as "10 x 10".)

In short. a *Vorstellung* or *conception* is a relatively independent conceptual structure in its own right and does not "refer to" or "stand for" something else. But — and to this we return later — it can very well be semantically linked to a word or larger piece of language.

The third use of "to represent" seems to be the least problematic. It is defined in my Concise Oxford Dictionary as "Fill the place of, be substitute or deputy for, be entitled to speak for,...". By and large it does not create semantic difficulties. It seems to be clear enough in most cases that the item that "represents" another in this way is explicitly designated or empowered to do so on specific occasions and in specifically limited activities.

Even so, it does at times give rise to confusion. When Caligula decided that his horse should "represent" him in the Roman Senate, when a tyrannical governor of Switzerland proclaimed that the people would have to greet his hat as though it were himself, or when nations decreed that a piece of cloth "embodied" their glory and should therefore be saluted, semantic and other conventions got out of hand. This usually happens when a purely symbolic" representative is turned into an "idol," i.e., a substitute imbued with inordinate power. No doubt such confusions are conceptual, too, but their import is primarily emotional, ideological, or political, and I prefer to disregard them here. With symbols, however, we must deal, but given the German words on which this disentanglement is based, this has to be the topic of the next section.

Bedeuten, in ordinary German, is the word for "to signify" and "to mean" as well as for "to denote". This ambiguity has the same confounding consequences as the ambiguity of "to mean" in English. The German word is used as indiscriminately as the English in sentences such as "These clouds mean rain" and "*Hund* means 'dog'." Thus, the difference between an inductive experiential relation and an arbitrary semantic relation is obscured and the floodgates of erudite obfuscation are opened.

At first glance, one might think that things are not quite as bad with "to represent", because in English it would sound odd to say: "These clouds represent rain". But if we look a little further, it soon gets messy. We come across phrases such as 'His negligence represented a threat to the project" and "This letter represents an insult", where "to represent" is used as though it were synonymous with "to constitute", "can be interpreted as", or simply the copula "to be". I could add that

these phrases represent the lowest level of semantic precision — and in doing so I would supply yet another current meaning of "to represent", namely "to be an example of". There is no end to the list of possible variations and, although some of them might throw interesting sidelights on the multifaceted conceptual structure of "representation", I do not intend to pursue their investigation here. This, indeed, is the reason why I put the "X" in the fourth of the initial sample sentences between quotation marks.

I want to consider only the kinds of item that have been deliberately chosen to *represent*, items such as the letters of the alphabet, words, symbols, graphs, and other artificial signs.³

Icons versus Symbols

There is a widespread confusion about two distinctions that have been made in the categorization of signs, symbols, and other semiotic items. The first is between *iconic* and *noniconic*, and it is analogous to the distinction mentioned in the context of iconic representations in a previous section. The second contrasts artificial signs with natural ones on the grounds that the former have to be deliberately chosen, whereas the latter arise out of the ordinary inductive inferences by means of which a cognitive subject organizes experience. The confusion was generated, at least in part, when the word "arbitrary", introduced to characterize the noniconic items, was slipped into the second distinction as a purported opposite to "natural" signs. Whereas it is unquestionably the case that artificial signs are always "arbitrary" in the sense that someone deliberately chose them (out of an infinite number of possibilities) to stand for something else, it is equally unquestionable that these artificial signs can be either iconic or noniconic. For instance, the now ubiquitous sign that features a crossed out cigarette is an artificial sign irrespective of the fact that it is iconic in that it depicts a cigarette.

Similarly, it was an arbitrary choice that instituted "X" as a symbol for "an unknown quantity" in mathematical notation. It so happens that it is also noniconic.

The coexistence of iconic representations and noniconic or symbolic signs goes back to about 30,000 B.C., the date of the first "representational" images of which we know today. They are statuettes of animals and human figures, so obvious that they were at once recognized as "iconic" representations when they were found in a German cave in the 1930s. Not so obvious was what Alexander Marshack's recent microscopic examination of artifacts has shown: These objects were used continuously for many years by their owners, who deliberately carved marks into them, presumably to record occurrences of some kind of event (Marshack, 1976).

A carefully made scratch, a straight line or angle, was thus chosen to "represent" an instance of a particular experience. Taken individually, there is nothing *iconic* about these marks. Noticing them, perceiving them, gives us not the slightest clue about what they were supposed to "represent". There is no analogy, no correspondence between experiencing the mark and whatever experience they were intended to refer to. They are truly symbolic, if by *symbolic* we want to indicate that

³ For the first distinction between "natural" and "artificial" signs see. for instance, Susanne Langer's (1948) *Philosophy in a New Key*, (p.58).

some item was arbitrarily chosen to stand for something else. I want to emphasize that it would be absurd to argue that such a mark should still be considered iconic because, being a single mark, it stands for a single experience. To be considered "a mark", whatever perceivable item one produces must be such that it can easily be isolated from the rest of the perceptual field (if this is not achieved, the item simply fails to function as a "mark"). Hence, the fact that marks, signs, or symbols must be perceived as unitary experiential items intended to refer to a segment of experience that, also, has been isolated from the rest of experience as a discrete and distinguishable piece is simply a prerequisite of marking, signing, symbolizing, and representing. Without it, no semiotic relation whatever could be established.⁴

Thus, one scratch by itself should not be taken as an iconic mark for oneness. The numerical iconicity, however, enters the moment two or more such marks are accumulated to indicate that the marked experience has been repeated so many times. In that case, the sequence of marks iconically represents the number of instances, irrespective of the fact that the marks, as such, give no indication as to what kind of experience was instantiated. Hence, a linear array of three designs, scratched into a deliberate iconic representation of a human female, will therefore be interpreted as someone's record of a repeated experience; but, there is no way of deciding whether they individually represent years the female survived, children she gave birth to, dragons slain in her honor, or anything else considered worth recording in the experience of the statuette's owner. Thus, already in the beginnings of human culture, we have examples of two kinds of representation, the iconic and the symbolic. Representations of number do, indeed, provide a complex illustration of what is iconic and what is not. Though the marks on the prehistoric finds and the three first numerals in the Roman system are noniconic with regard to the kind of item they are intended to record, they are iconic in the context of numeration, whereas the Roman numerals "V", "X", "L", etc., and the Arabic numerals are not.5

Both refer to another item, but icons do so by means of sensorimotor similarity whereas arbitrary signs and symbols refer by assignation or social convention. Anyone may infer that a *fleur de lys* is a stylized picture of a lily; no one could infer (by looking at it) that it is the symbol of the Kings of France. This second connection is arbitrary because it has nothing whatever to do with the character of specific perceptual or sensorimotor operations.

With regard to icons, Piaget's distinction between the "figurative" and the "operative" would seem to be of some importance. Number is not a perceptual but a conceptual construct; thus it is operative and not figurative. Yet, perceptual arrangements can be used to "represent" a number figuratively. Three scratches on a

⁴ Note that this. of course. does not preclude that marks, signs, symbols. etc., can be composites containing any number of elements; it merely means that whatever arrangement of elements is selected and intended to stand for something else must be such that, *in the given perceptual context* it is likely to be isolated in the perceptual field and taken as a discrete, coherent item.

⁵ The fact that Arabic numerals were derived from iconic signs by gradual modification does not make them less symbolic today. What matters is that we and our children do not perceive them as composed of countable items and have to be told what they mean.

prehistoric figurine, for instance, can be interpreted as a record of three events. In that sense they may be said to be "iconic" — but their iconicity is indirect. They do not depict "threeness", they merely provide the beholder with an occasion to carry out the conceptual operations that constitute threeness (von Glasersfeld, 1981, 1982). Carrying out these operations does not involve *reference* to some prior sensorimotor item or elements of such items — it is the operating itself that each time constitutes the abstract conception of threeness.

This difference between figurative icons that refer to something else and operative icons that simply trigger the construction of a specific abstract conception is, I believe crucial in sorting out the kinds of "representation" Jim Kaput (1984) cited from Palmer (1978).

An analogous distinction must be made in the case of symbols. On the one hand, there are symbols that refer to figurative items or sensorimotor situations, such as the King of France or the act of smoking; on the other, there are symbols that do not refer to sensorimotor experience at all but are merely indicators that a certain conceptual operation is to be performed. I would call this second category operative symbols and would list among them not only number words, numerals, and mathematical signs, such as "+", "—", and "=", but also prepositions, conjunctions, and certain other words whose interpretation does not depend on the recall of sensorimotor experiences but requires the construction of some operative conceptual relation.

Final Remarks

A great deal more should be said about the category of *mental representations* I have called conceptions. It is here that one's basic theory of knowledge plays perhaps the most decisive role. From my radical constructivist point of view, all the constructs by means of which we assimilate the flow of experience into our "order" or Weltbild fall into this category. But I have already exceeded the allotted space and much of what, to me, seems relevant to the discussion of this type of mental representation is implicit in other papers I have written.⁶ In any case, these pages should have made it clear that, from the constructivist perspective, it makes no sense to think of mental representations as any kind of *Darstellung* or depiction of ontological reality. From my point of view, the proponents of the various forms of realism throughout the centuries have failed to come up with a viable theory of representation. The recent introduction of the spurious term *information* seems, for the moment, to have revived the old illusion that the gap between conceptual constructs and the ontological world can be bridged. But there is little benefit in speaking of "representations" or, indeed, "translation", where, as Kant's *Critique* has so irrefutably shown, there is no logically possible access to what they are supposed to represent.

⁶ See, for instance, my Introduction to Radical Constructivism (1984) and On the Concept of Interpretation (1983).

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