Revised version of: The “truth” about Pythagoras, *Problem Solving,* 1981, *3* (5/6), 1–11. 071

How Did Pythagoras Do It?

Although geometry had been my favorite subject in high school, and in 1937 I was nearly halfway to a never finished doctorate in mathematics, no one had ever told me that Pythagoras had probably proved his theorem (if, indeed, it was he who proved it) by an intuitively much more convincing method than the algebraic development we had been taught in school. I happened upon that revelation in “Young Archimedes”, a short story by Aldous Huxley, which, unlike algebra, made an indelible impression on me.[1]

I found it once more in print 40 years later in Ackermann’s *Theories of Knowledge*.[2] Seeing it neatly drawn in black and white helped me formulate a question from my vague feeling of wonder: How could Pythagoras ever have thought of that proof if he didn’t already have an idea of what he wanted to prove? It’s plausible that, in some context or other (or even just doodling) one might draw a square and place two crossed perpendiculars in it, so that the area is divided into two equal rectangles and two unequal squares. It’s also conceivable that one might add diagonals to each of the rectangles, still without the presupposition of any particular plan (Fig.1). Then. however, to draw a second square and to arrange the four triangles in the particular way necessary to prove a theorem one does not yet know (Fig.2)—that is something that seems very unlikely to be purely fortuitous.

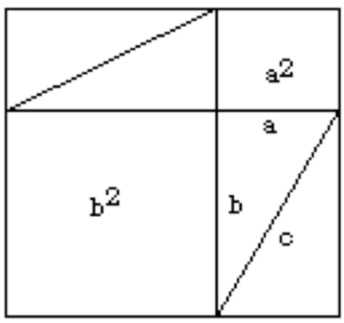

Fig. 1

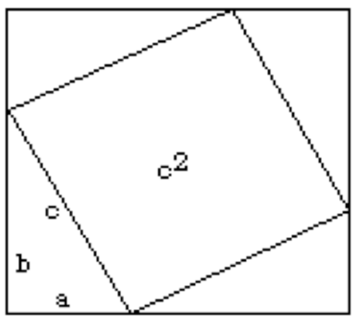

Fig. 2

Once the second diagram has been drawn, of course, the Pythagorean theorem is plainly demonstrated—but the beautiful “obviousness” of that demonstration cannot explain why the diagram was conceived in the first place. That question may be answered by a decorative pattern that was quite common long before Pythagoras’ time and that was later used for a different purpose by Socrates in Plato’s Menon. The pattern goes back to the Middle Kingdom in Egypt, as far as 2000 B.C. In the first three books on Greek art and architecture that I consulted, I found it half a dozen times, for instance as a lattice in the facade of the Alcmeonidae temple at Delphi (510 B.C.)[3] and as design on the “printed” dress of a figure on a Greek vase that dates from 570 B.C.[4] It’s a simple pattern and seems to have been used on many things in many places, perhaps even in a tiled floor of a house Pythagoras lived in, on the island of Samos, where he was born. It is not much of a conjecture to assume that Pythagoras must have seen it. Being a man of imagination, he would see such a design in more than one way: he would play with the different possible ways of relating the geometric elements within the composition. In so doing he could not help but discover his theorem.

This is the pattern:

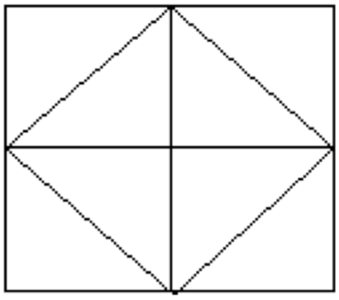

Fig. 3

If you first see the pattern as in Fig.4 and at a subsequent moment as in Fig.5

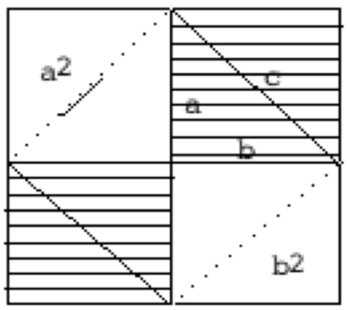

Fig. 4

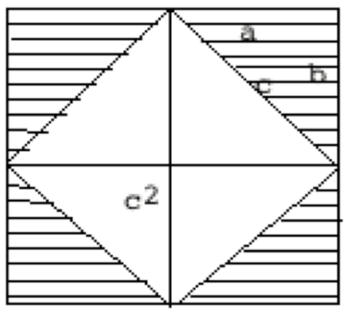

Fig. 5

you have the visual proof that a2 + b2 = c2, in the case of an isosceles right-angled triangle. Pythagoras, who was very interested in regular shapes such as squares and triangles, could have discovered this any day after lunch, when the sun was shining on his tiled floor. But Pythagoras was also interested in more general abstract regularities. Hence he would immediately want to know whether or not his discovery would hold for other right-angled triangles—and it was then that he may have resorted to drawing the two diagrams of Fig.1 and Fig.2, because in one diagram one cannot readily accommodate both arrangements of the triangles if they are not isosceles.

If this story of how Pythagoras may have found his proof were told in school, I believe he and his theorem would be a good deal more interesting to students. It might also serve as illustration of a far more generally useful idea: One can often solve a problem simply by looking at things in more than one way.

Footnotes

1. Huxley, Aldous. *Little Mexican*. London Chatto & Windus, 1924.
2. Ackermann, Robert. *Theories of Knowledge*. New York: ~IcGraw-Hill, 1965.
3. Pekaros, B.C. *Delphi*. Athens (Greece): Hesperos. 1971.
4. Boardman, J. *Greek art*. New York: Praeger, 1969.

Acknowledgment

I thank Patricia Graham-Collier for leading me straight to the relevant books on antique art.
