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Introduction

[to *Radical Constructivism in Mathematics Education*]

Mathematics is the science of *acts without things* – and through this, of things one can define by acts. Paul Valéry[1]

The essays collected in this volume form a mosaik of theory, research, and practice directed at the task of spreading mathematical knowledge. They address questions raised by the recurrent observation that, all too frequently, the present ways and means of teaching mathematics generate in the student a lasting aversion against numbers, rather than an understanding of the useful and sometimes enchanting things one can do with them.

Parents, teachers, and researchers in the field of education are well aware of this dismal situation, but their views about what causes the wide-spread failure and what steps should be taken to correct it have so far not come anywhere near a practicable consensus.

The authors of the chapters in this book have all had extensive experience in teaching as well as in educational research. They approach the problems they have isolated from their own individual perspectives. Yet, they share both an overall goal and a specific fundamental conviction that characterized the efforts about which they write here. The common goal is to find a better way to teach mathematics. The common conviction is that knowledge cannot simply be transferred ready-made from parent to child or from teacher to student but has to be actively built up by each learner in his or her own mind.

My purpose in compiling this book, and the purpose of the authors who have contributed its chapters, is not to make yet another move in the theoretical philosophical debate about constructivism. The book is intended to clarify how the didactic attitude changes when the constructivist theory of knowing is put into practice, and what results have been attained in doing so. Rather than forecasts of what might be achieved in the future, the papers assembled here report on experiments and implementations that have actually been carried out.

The individual pieces speak for themselves and require no amplifications. They cover a wide range of educational efforts: Elementary school (Cobb, Steffe, van den Brink), high school (Balacheff, Richards), undergraduate instruction (Confrey, Kaput, Konold, Lochhead), and teacher preparation (Underhill). Consequently they deal with different areas of mathematics, and it may be useful to anticipate and spell out here some of the principles that underlie the constructivist approach common to all the contributions.

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Language frequently creates the illusion that ideas, concepts, and even whole chunks of knowledge are transported from a speaker to a listener. This illusion is extraordinarily powerful because it springs from the belief that the meaning of words and phrases is fixed somewhere outside the users of the language. Perhaps the best way to dismantle the illusion is to remember or reconstruct how one came to form the meanings of words and phrases when one was acquiring language in the first place. Clearly it could only be done by associating bits of language one heard with chunks of one's own experience – and no one's experience is ever exactly the same as another person's. Thus, whatever another says or writes, you cannot but put your own *subjective* meanings into the words and phrases you hear. Given that we live in a community of other language users, our subjective meanings tend, of course, to become *intersubjective*, because we learn to modify and adapt them so that they fit the situations in which we interact with others. In this way we manage to achieve a great deal of compatibility. But to prove compatible, individual meanings do not have to be identical. Indeed, throughout our lives we now and then discover that the meaning we have associated with a certain word is not yet quite compatible with the use others make of that word.[2] This may serve to remind us – especially when we act as teachers that new concepts and new knowledge cannot simply be passed to another person by talk, because each must abstract meanings, concepts, and knowledge from his or her own experience.

This does not mean that language cannot be used to *orient* (Maturana, 1980) students towards certain experiences and certain mental activities such as abstracting; but it does mean that we can never rely on language to "convey" knowledge as though it were something like food that can be handed from one to another.

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The notion that knowledge is the result of a learner's activity rather than of the passive reception of information or instruction, goes back to Socrates and is today embraced by all who call themselves "constructivists." However, the authors whose work is collected here, constitute the radical wing of the constructivist front. They have taken seriously the revolutionary attitude pioneered in the 1930s by Jean Piaget, the Swiss founder of cognitive psychology. This attitude is characterized by the deliberate redefinition of the concept of knowledge as an *adaptive function*. In simple words, this means that the results of our cognitive efforts have the purpose of helping us to cope in the world of our experience, rather than the traditional goal of furnishing an "objective" representation of a world as it might "exist" apart from us and our experience.[3] This attitude has much in common with the pragmatist ideas proposed by William James and John Dewey at the beginning of this century. It is *radical* because it breaks with the traditional theory of knowledge and it has profound consequences for parents, teachers, and researchers whose objective is to generate particular ways of acting and thinking in children and students. Steedman's essay grounds the changed attitude in the philosophy of science and the other contributors illustrate some of the consequences for the practice of education.

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When teachers look at a student in their class, they see that student in a complex environment. There is the physical environment of the desk (or whatever piece of furniture the student occupies), the classroom, and the land-or townscape around the school building, part of which might be visible through the window; there is the social environment of the classmates, the teachers themselves, and the student's family, which may to some extent be known to the particular teachers, and there is also the didactic environment of the school books, the educational props, the curriculum, and, more important but less perceivable, the teachers' views and beliefs as to how education should be achieved.

From the naive commonsense perspective, the elements that form this complex environment belong to a *real* world of unquestionable objects, as real as the student, and these objects have an existence of their own, independent not only of the student but also of the teacher.

Radical Constructivism is a theory of *knowing* which, for reasons that had nothing to do with teaching mathematics or education, does not accept this commonsense perspective.[4]

Instead, it takes seriously the no less venerable suggestion that what a teacher sees when he or she looks at a student is necessarily part of the particular teacher's experience and, as such, the result of the particular teacher's ways and means of perceiving and conceptualizing what is being perceived. Thus, the teacher's view of the student, the classroom and its furniture, the surroundings of the school house, the classmates, the student's family, the school books and other educational material, the curriculum, and, needless to say, the teacher's views about education, are and cannot be anything but the particular teacher's own experiential world.

Superficial or emotionally distracted readers of the constructivist literature have frequently interpreted this stance as a denial of "reality."[5] If everyone had a different experiential world, they tend to argue, we could not agree on anything and, above all, we could not communicate. There is not much wrong with that argument, but the fact that we *do* agree on certain things and that we *can* communicate does not prove that what we experience has *objective reality* in itself. If two people or even a whole society of people look through distorting lenses and agree on what they see, this does not make what they see any more *real* – it merely means that on the basis of such agreements they can build up a consensus in certain areas of their subjective experiential worlds. Such areas of relative agreement are called "consensual domains," and one of the oldest in the Western world is the consensual domain of numbers. The certainty of mathematical "facts" springs from mathematicians' observance of agreed-on ways of operating, not from the nature of an objective universe. It is a shared belief of our authors that establishing a consensual domain that comprises the instructor as well as the learners is a prerequisite of teaching.

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From an educator's point of view, one of the most important features of radical constructivism is the sharp distinction it draws between teaching and training. The first aims at generating *understanding*, the second at *competent performance*. The fifty years of behaviorist hegemony in American psychology all but succeeded in eliminating that distinction. The sophisticated techniques developed by Professor Skinner and his followers have shown that in many tasks almost flawless performance can be achieved by the methodical management of stimuli and reinforcement. Yet, if our purpose is to spread mathematical knowledge, these techniques are inappropriate because the "competence" they engender has been deliberately cut loose from mental operations and understanding.[6] To know mathematics is to know how and why one operates in specific ways and not in others, how and why the results one obtains are derived from the operations one carries out.

The focus on understanding leads to several other considerations. As long as students merely strive to find answers that are "right" because they satisfy the teacher, the students may learn to get through class more or less painlessly, but they will not learn mathematics. The early dialogs in John Richards' chapter are a blatant example of this aberration. When that spurious goal has been eliminated, another point comes to the fore: Students who are trying to solve a problem, rather than please the teacher, do not answer randomly. What they produce makes sense to *them* at the time, no matter how little sense it might make to the more advanced mathematician. And it is this sense that the teacher must try to understand if he wants to find ways and means of modifying the students' thinking. Several of our authors emphasize this aspect of the teacher's task. Steffe calls it *constructing a model of the student's concepts and operations* and we all consider it indispensable as a working hypothesis for the teacher who intends to guide the student towards the kind of mathematics to be learned.

However, the models we construct of other people's ideas and mental operating, regardless of whether we are concerned with their politics or their mathematics, are necessarily hypothetical because we have no direct access to what goes on inside other people's heads. Nevertheless we can test the *viability* of our models of students' thinking by creating situations in which these ways of thinking would be likely to produce certain observable results. This is no different in principle from physicists who set up experiments to test their models of unobservable sub-atomic particles. The traditional psychologists' objection to mental explanations, thus, is nothing but the remnant of a superannuated early 19th-century view of "science."

Indeed, if we are serious in maintaining that learning aims at *understanding*, we should avoid the term "mathematical literacy" that has recently become a cliché of mathematics education. Research on reading was for a long time hampered by the shallow notion that literacy was simply a matter of recognizing written or printed letters and being able to reproduce words vocally. Only when comprehension on the conceptual level was taken into account, did the research in that area make some progress. Similarly, to recognize a sequence of spoken or written mathematical symbols as the stimulus for certain conventional operations may be a prerequisite of conceptual understanding, but it is no more an indication of comprehension than the ability vocally to produce the sentence "time and space form a continuum" would indicate an *understanding* of Einstein's theory of relativity.

The models we construct of other people's thought processes can, of course, never be anything but *our* constructs built of components that are accessible to *us*. But this inescapable limitation does not mean that we cannot use our cognitive ability to construct a view and an interpretation of the given situation that turns out to be more compatible with what we perceive of the other's actions and reactions. Not surprisingly, this effort may lead the teacher to a new and fuller understanding of the subject area (Confrey, Underhill). Yet as long as teachers cling to the traditional conviction that the solutions to mathematical problems are not only obvious but also unique, and that the failure to see them is due either to stupidity or a lack of application, they have little if any incentive to investigate whatever sense the students might make of a problem and their tentative solutions.

As an important corollary, the teacher's attempts to understand the individual student's approach to a "problem" generate a climate of positive social interaction. Genuine interest in how they think shows the students, better than any verbal affirmation, that they are being taken seriously; and this, in turn, enhances their courage to try and openly discuss new paths of whose outcome they are still uncertain. There is no more efficient way to generate the kind of *reflection* that is necessary for conceptual advancement (Cobb, Confrey, Kaput, Richards, Steffe, Underhill, van den Brink).

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Although Piaget, for more than five decades, maintained that all "operative" knowledge (i.e., the know-how concerning mental operations) was the result of reflection, psychologists who claimed to be strict empiricists took great pains to avoid any mention of the term. This is odd, because one of the few things Locke, Berkeley, and Hume, the founders of Empiricism, agreed on, was that all knowledge comes either from the senses or from reflection, which Locke had defined as the mind's ability to ponder its own operations. (To avoid embarrassment about not having acknowledged the role of reflection for so long, the psychological establishment is now rediscovering at least parts of it under the name of "metacognition.")

From the constructivist point of view, there can be no doubt that reflective ability is a major source of knowledge on all levels of mathematics. That is the reason why nearly all the contributors to this volume consider it important that students be led to talk about their thoughts, to each other, to the teacher, or to both. To verbalize what one is doing ensures that one is examining it. And it is precisely during such examination of mental operating that insufficiencies, contradictions, or irrelevancies are likely to be spotted (Balacheff, Cobb, Confrey, Lochhead, Kaput, Konold, Richards, van den Brink).

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A thinking subject has no reason to change his or her way of thinking as long as there is no awareness of failure. But there are many shades and forms of failure, and the one most common in conventional schools – when the teacher declares that the student is "wrong" – is the least effective in bringing about a change in the student's way of thinking. Far more powerful is the failure to gain the agreement of peers who are struggling with or have solved the same problem. As Piaget reiterated many times, after infancy the most frequent cause of *accommodation* (change in a way of operating or acting) arises in social interaction when the individual's ways and means turn out to be in some sense insufficient in comparison to the ways and means of others (Piaget, 1967; cf. also Rowell, 1989).

Analogously, a thinking subject has no occasion to feel the intellectual satisfaction of having solved a problem, if the solution did not result from his or her own management of concepts and operations but was supplied from outside. Here again the behaviorist notion of social approval as the prime reinforcement has helped to disorient schooling practice. It is not that a teacher's approval and pat on the head have no effect on the student, but the effect is to strengthen the student's inclination to please the teacher rather than to build up *understanding* of the conceptual *area* in which the task was situated. Thus students are prevented from experiencing the rewarding elation that follows upon having found *one's own way and recognizing it as a good way*. If students are not oriented or led towards autonomous intellectual satisfaction, we have no right to blame them for their lack of proper *motivation* The motivation to please superiors without understanding why they demand what they demand, may be required in an army in an institution that purports to serve the propagation of knowledge, it is out of place.

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The empirical investigations and the applications of constructivist ideas in teaching and learning situations were carried out and are here reported by highly original researchers, each of whom has developed an individual style. The theoretical underpinnings, however, are largely compatible if not the same. Radical Constructivism, I want to emphasize, is a theory of *active knowing* , rather than a traditional theory of knowledge or epistemology. From this standpoint, as Piaget maintained fifty years ago, *knowledge serves to organize experience*, not to depict or represent an experiencer-independent reality.

One of the main sources of the constructivist theory was an analysis of linguistic communication that exposed the subjective roots of the *meanings* in terms of which each individual interprets the language heard or read. For educators, this reinforces the intuition that the student's experiential reality is not the same as the teacher's, and it makes explicit the fact that, because language does not "transport" knowledge, it must and can be used to *orient* a student's own conceptual construction.

The claim that thinking individuals create for themselves individual realities does not imply, as critics of constructivism tend to assert, the denial of an external reality; it claims no more than it literally says, namely that individuals organize their experience in their own subjective ways.

The constructivist theory, consequently, clearly distinguishes *training* from *teaching*. The former may lead to the replication of a behavioral response, the latter aims at generating *autonomous conceptual understanding*.

Teachers have a better chance to modify a student's conceptual structures and understanding if their interventions are informed by a hypothetical model of the student's present ideas. By reinforcements or threats, they may get the student to repeat anything they say, but understanding cannot be forced in this way. Moreover, the mere acceptance of the solution of a problem that is forced on them, is not likely to give students a taste of the kind of intellectual satisfaction that would generate a genuine motivation to understand more and to delve further into the problem area.

In contrast, leading students to discuss *their* view of a problem and their own tentative approaches, raises their self-confidence and provides opportunities for them to reflect and to devise new and perhaps more viable conceptual strategies. This is one among several reasons why all the contributions to this volume stress the importance of the social interaction and the social climate the teacher establishes in the classroom.

As a constructivist, I am fully aware of the fact that the introductory notes I have presented here cannot determine the interpretation of the essays that follow. But they may orient the reader to look for some of the things that provided the original impetus for generating this volume.

E.v.G.
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Amherst, Massachusetts

Notes

1. Valéry, who was a mathematician as well as a poet, wrote this in his notebook in 1935 (cf. Valêry, 1974: p. 811). Already in 1903 had he jotted down that "mathematics is the description of mental operations insofar as they can be exactly apperceived" (*ibid.*, p. 783).
2. If we are purists, we go to a dictionary to find the *correct use* – and we tend to forget that what we read there is still subject to *our* interpretation in terms of our own subjective experience.
3. For a full exposition of the radical constructivist theory of knowing see von Glasersfeld, 1978,1981, 1985).
4. Starting from somewhat different considerations, Heinrich Bauersfeld has come to a similar conclusion and speaks of "subjective domains of experience" (1983).
5. E.g. Kilpatrick (1987), Kitchener, (1986).
6. The universally deplored decline of critical thinking ability is but one of the sad by-products of the exclusive emphasis on behavioral competence.

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