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Aspects of Radical Constructivism and its Educational Recommendations

In the context of theories of knowledge, the name "radical constructivism" refers to an orientation that breaks with the Western epistemological tradition. It is an unconventional way of looking and therefore requires conceptual change. In particular, radical constructivism requires the change of several deeply rooted notions, such as knowledge, truth, representation, and reality. Because the dismantling of traditional ideas is never popular, proponents of radical constructivism are sometimes considered to be dangerous heretics. Some of the critics persist in disregarding conceptual differences that have been explicitly stated and point to contradictions that arise from their attempt to assimilate the constructivist view to traditional epistemological assumptions. This is analogous to interpreting a quantum-theoretical physics text with the concepts of a 19thcentury corpuscular theory.

It may be useful, therefore, to reiterate some points of our "postepistemological" approach,[1] so that our discussion might have a better chance to start without misinterpretations.

No Exit from Subjectivity

Radical constructivsm is an attempt to develop a theory of knowing that is not made illusory from the outset by the traditional assumption that the cognizing activity should lead to a 'true' representation of a world that exists in itself and by itself independently of the cognizing agent. Instead, radical constructivism assumes that the cognizing activity is instrumental and neither does nor can concern anything but the experiental world of the knower. This experiential world is constituted and structured by the knower's own ways and means of perceiving and conceiving, and in this elementary sense it is always and irrevocably subjective. It is the knower who segments the manifold of experience into raw elementary particles, combines these to form viable 'things', abstracts concepts from them, relates them by means of conceptual relations, and thus constructs a relatively stable experiential reality. The viability of these concepts and constructs has a hierarchy of levels that begins with simple repeatability in the sensory-motor domain and turns, on levels of higher

[1] I owe this expression to Nel Noddings, who used it in a review of one of my papers.

abstraction, into operational coherence, and ultimately concerns the non-contradictoriness of the entire repertoire of conceptual structures.

The statement that the construction of the experiential world is irrevocably subjective, has been interpreted as a declaration of solipsism and as the denial of any 'real' world. This is unwarranted. Constructivism has never denied an ulterior reality; it merely says that this reality is unknowable and that it makes no sense to speak of a representation of something that is inherently inaccessible.

The insistence on the subjectivity of the experiential world has also led some critics to the rash conclusion that radical constructivism ignores the role of social interaction in the construction of knowledge. This, too, is a misinterpretation, and a rather thoughtless one. If one begins with the assumption that all knowledge is derived from perceptual and conceptual experience, one in no way denies that 'others' and 'society' have an influence on the individual's cognitive constructing; but one will remain aware of the fact that these others and the society they constitute 'exist' for the individual' subject only to the extent to which they figure in that individual's experience – that is to say, they are for each subject what he or she perceives and conceives them to be.

In contrast, those who call themselves social constructionists, tend to introduce the social context as an ontological given. They are, of course, free to do so; but it does not entitle them to fault another school of thought that endeavors to build a theory of knowing without ontological givens or other metaphysical assumptions. This was seen quite clearly sixty years ago by Alfred Schu¨tz (1932) when he referred to "the immensely difficult problems that are tied to the constitution of the thou in each individual's own subjectivity" and added a few lines later that "...such analyses belong to the general theory of knowledge and thus mediately to the social sciences" (p. 138). Radical constructivism is indeed intended as a theory of knowing and therefore is obliged to attempt an analysis of how the thinking subject comes to have others in his or her construction of the experiential world (cf. von Glasersfeld, 1986).

Some Salient Characteristics

From the radical constructivist point of view, the basic ideas concerning the questions what is knowledge and how do we come to have it, can be summarized as follows.

No philosopher in the course of the last 2500 years has been able to demolish the sceptics' logical arguments that the real world, in the sense of ontological reality, is inaccessible to human reason. In view of this impasse,, constructivism, like the pragmatists at the beginning of our century, suggests that we change the concept of knowledge. The pragmatists, however, remained attached to a metaphysical if not material form of realism. Instead, constructivism goes back to Vico, who considered human knowledge a human construction that was to be evaluated according to its coherence and its fit with the world of human experience, and not as a representation of God's world as it might be beyond the interface of human experience. Constructivism drops the requirement that knowledge by 'true' in the sense that it should match an objective reality. All it requires of knowledge is that it be viable, in that it fit into the world of the knower's experience, the only 'reality' accessible to human reason.

With regard to the cognitive construction, we follow the two pioneers of conceptual analysis, Jean Piaget and Silvio Ceccato. That is to say, we attempt to build plausible models of how, by means of reflection and abstraction, viable concepts could be derived from subjective experience.

This change of view has consequences not just for a few traditional philosophical beliefs but for almost everything one habitually thinks about acts of knowing and knowledge resulting from them. Here I want to mention only two cases in point.

Inherent in radical constructivism is the realization that no knowledge can claim uniqueness. In other words, no matter how viable and satisfactory the solution to a problem might seem, it can never be regarded as the only possible solution. (Note that this does not contradict the observation that, for instance in mathematics, solutions are often fully determined by the operations one carries out to find them.)

The second is Leo Apostel's admonition that "a systems should always by applied to itself" (Inhelder et al., 1977, p. 61). In our case, this leads to the conclusion that radical constructivism cannot claim to be anything but one approach to the age-old problem of knowing. Only its application in contexts where a theory of knowing makes a difference can show whether or not it can be considered a viable approach.

Concerning Education

Indeed, here at ICME-7, we are not primarily concerned with philosophical questions, but rather with applications to the teaching of mathematics. In this regard, let me emphasize that, though we have promising beginnings (cf. Steffe et al., 1983,1988; Steffe, 1991; von Glasersfeld, 1981 & in press), the enormous task of analyzing the basic conceptual steps in the construction of mathematics has barely begun.

Teachers at all levels, from elementary school to post-graduate instruction, have to rely on the use of language, and textbooks can not do without it. Yet, in my experience, few language users have given much thought to the question how linguistic communication is supposed to work.

In everyday circumstances, where most of what we say and others say to us, does not give rise to obvious misinterpretation, we usually assume that the meaning of words and sentences is the same for all speakers of the particular language. If there are differences, they seem to be insignificant. I have shown elsewhere that, even in the case of the most ordinary objects, the notion of 'shared meaning' is strictly speaking an illusion. This is so because we associate the sounds we come to isolate as 'words' not with things but with our subjective experiences of things – and though subjective experiences may be similar for different subjects, they are never quite the same (von Glasersfeld, 1990).

The Making of Abstractions

Here, however, we are concerned with mathematics teaching and thus not with sensory items but with concepts that are abstracted from mental operations. In the case of ordinary sensory objects, the individual gradually learns by interacting in practical situations with other speakers of the language, to adjust his or her meanings so that they become more or less compatible with those current in the community. In the case of abstract items, however, it is far more difficult to achieve this social adequation, because the occasions where conceptual discrepancies might come to the surface are few and far between. Hence, in order to teach abstract notions, it is indispensable for the instructor to generate experiential situations for the students to make the necessary abstractions. In order to foster such abstractions, the teacher must be successful in establishing with the students a common language, i.e., a language of carefully negotiated and coordinated meanings or, as Maturana has called it, a consensual domain (Maturana, 1980; Richards, 1991).

Mathematics is the result of abstraction from operations on a level on which the sensory or motor material that provided the occasion for operating is disregarded. In arithmetic this begins with the abstraction of the concept of number from acts of counting. Such abstractions cannot be given to students, they have to be made by the students themselves. The teacher, of course, can help by generating situations that allow or even suggest the abstraction. This is where manipulables can play an important role, but it would be naive to believe that the move from handling or perceiving objects to a mathematical abstraction is automatic. The sensory objects, no matter how ingenious they might be, merely offer an opportunity for actions from which the desired operative concepts may be abstracted; and one should never forget that the desired abstractions, no matter how trivial and obvious they might seem to the teacher, are never obvious to the novice.

The same can be said about the use of multiple representations (Kaput, 1991; Gerace, in press). In learning to switch from one representation to another, an act of reflective abstraction may focus on what it is that appears to remain the same. But this abstraction is, again, not automatic, and it may well be precluded if the switch is explicitly presented as the simple exchange of two equivalent items. The point is that the representations are different, but an operative concept or conceptual relation they embody is considered the same.

Meaning and Misconceptions

In contrast, the need for an experiential basis for the abstraction of concepts is often overlooked, because of the formalist myth that all that matters in mathematics is the manipulation of symbols. This ignores the fact that spoken words or marks on paper are symbols only if one attributes to them something they symbolize, i.e., a meaning – and meaning is always conceptual. As Hersh said: “Symbols are used as aids to thinking just as musical scores are used as aids to music. The music comes first, the score comes later” (1986, p. 19). There is little point in teaching a score to students who have no music to relate to it. In school, however, mathematical symbols are often treated as though they were self-sufficient and no concepts and mental operations had to accompany them; but when students are only trained to manipulate marks on paper it is small wonder that few of them ever come to understand the meaning of what they are doing and why they should do it.

Because there is no way of transferring meaning, i.e., concepts and conceptual structures, from one head to another, teachers, who have the goal of changing something in students’ heads, must have some notion of what goes on in those other heads. Hence it would seem necessary for a teacher to build up a model of the student’s conceptual world (see von Glasersfeld & Steffe, 1991).

From the constructivist perspective, it is not helpful to assume (maybe on the basis of “wrong” answers) that a student’s ideas are simply misconceptions that have to be replaced by the conceptions that are considered correct by mathematicians, physicists, or other experts. In order to become operative in a student’s thinking, a new conception must be related to others that are already in the student[‘s repertoire. No doubt there are several ways of establishing such relationships, but the simplest and most efficacious arises when the new structure is built out of elements with which the students are familiar. In other words, students must be shown that there are elements in their experience that can be related differently from the way they habitually relate them. To make such changes desirable to students, they must be shown that the new way provides advantages in a sphere of living and thinking that reaches far beyond passing exams and getting good grades.

Besides, when a student has struggled to find an answer to a given problem, it is not only boorish but also counterproductive to dismiss it as “wrong”, even if the teacher then shows the “right” way of proceeding. Such disregard for an effort made inevitably demolishes the student’s motivation. Instead, a wiser teacher will ask the student how he or she came to the particular answer. In the majority of cases, the student, in reviewing the path (i.e. reflecting upon the operations carried out), will either discover a hitch or give the teacher a clue to a conceptual connection that does not fit into the procedure that is to be learned. The first is an invaluable element of learning: it provides students with an opportunity to realize that they themselves can see what works and what does not. The second provides the teacher with an insight into the student’s present way of operating and thus with a clearer idea of where a change might be attempted.

To end this brief list of recommendations, let me repeat a rather unpopular point. From the constructivist perspective, whatever one intends to teach must never be presented as the only possible knowledge – even if the discipline happens to be mathematics. Indeed it should be carefully explained that a fact such as “2 + 2 = 4” may be considered certain, not because it was so ordained by God or any other extra-human authority, but because we come to construct units in a particular way and have agreed on how they are to be counted.

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